OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2602/1

Pure Mathematics 2

Monday **20 MAY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

HN/7

[Turn over

1 (a) Given that $y = \left(1+\sqrt{x}\right)^2$, show that $\dfrac{dy}{dx} = 1 + \dfrac{1}{\sqrt{x}}$. [3]

(b) Express $2\log_{10}x + \log_{10}3$ as a single logarithm.

Hence, or otherwise, solve

$$2\log_{10}x + \log_{10}3 = \log_{10}75.$$ [4]

(c) Using a suitable substitution, or otherwise, find $\displaystyle\int \frac{x}{(x^2-1)^3}\,dx$. [4]

(d) Evaluate $\displaystyle\int_1^2 \frac{x^2+2}{x}\,dx$. [4]

2

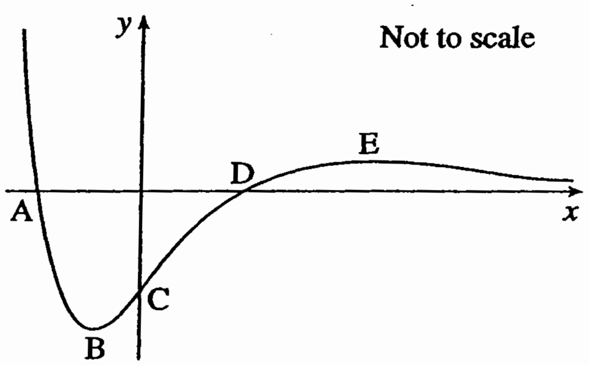

Fig. 2

Fig. 2 shows a sketch of the graph of $y = (x^2 - 3)e^{-x}$. The graph crosses the x-axis at points A and D and the y-axis at C. Points B and E are stationary points on the curve.

(i) Find the coordinates of the points A, C and D. [4]

(ii) Show that $\dfrac{dy}{dx} = -(x^2 - 2x - 3)e^{-x}$. [3]

(iii) Deduce that the x-coordinates of the points B and E are -1 and 3 respectively, and find the corresponding y-coordinates. [5]

(iv) Copy Fig. 2, and mark clearly the positions of two points of inflection.

You are given that $\dfrac{d^2y}{dx^2} = (x^2 - 4x - 1)e^{-x}$. Deduce from this result that there are exactly two points of inflection. [3]

3 The height h metres of a species of pine tree t years after planting is modelled by the equation

$$h = 20 - 19 \times 0.9^t.$$

(i) What is the height of the trees when they are planted? [1]

(ii) Calculate the height of the trees after 2 years, and the time taken for the height to reach 10 metres. [5]

The relationship between the market value £y of the timber from the tree and the height h metres of the tree is modelled by the equation

$$y = ah^b,$$

where a and b are constants. Fig. 3 shows the graph of $\ln y$ plotted against $\ln h$.

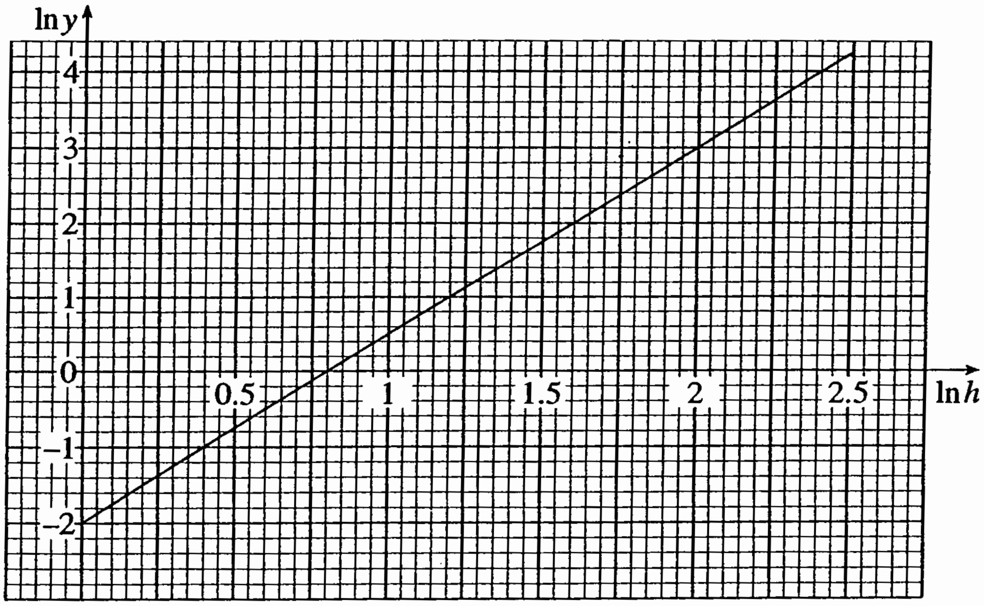

Fig. 3

(iii) Use the graph to calculate the values of a and b. [5]

(iv) Calculate how long it takes to grow trees worth £100. [4]

[Turn over

4 **(a)** In a race, skittles $S_1, S_2, S_3, \ldots$ are placed in a line, spaced 2 metres apart. Susan runs from the starting point O, b metres from the first skittle. She picks up the skittles, one at a time and in order ($S_1, S_2, S_3, \ldots$), returning them to O each time (see Fig. 4).

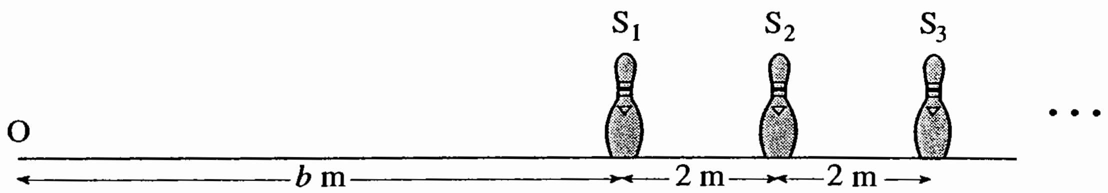

Fig. 4

(i) Show that the total distance Susan runs in a race with 3 skittles is $6(b + 2)$ metres. [1]

(ii) Show that the total distance she runs in a race with n skittles is $2n(b + n - 1)$ metres. [3]

(iii) With $b = 5$, the total distance she runs is 570 metres. Find the number of skittles in this race. [3]

(b) A geometric progression is defined by

$$u_k = 3 \times 1.25^{-k}, \qquad k = 1, 2, 3, \ldots .$$

(i) Calculate u_1, u_2 and u_3. What is the common ratio of the geometric progression? [4]

(ii) Calculate $\sum_{k=1}^{20} u_k$. [2]

(iii) Find the sum to infinity of the geometric progression. [2]

Mark Scheme

MEI Pure Mathematics 2 2602/01 June 2002
Mark Scheme (final for publication)

1(a) $y = (1+\sqrt{x})^2$ $\Rightarrow \dfrac{dy}{dx} = 2(1+\sqrt{x}).\ \dfrac{1}{2}x^{-1/2}$ $= \dfrac{1+\sqrt{x}}{\sqrt{x}} = 1 + \dfrac{1}{\sqrt{x}}$ *	M1 B1 E1	Chain rule (Consistent with their du/dx) $\dfrac{1}{2}x^{-1/2}$ Must show some working
or $y = (1+\sqrt{x})^2 = 1 + 2\sqrt{x} + x$ $\Rightarrow \dfrac{dy}{dx} = 2.\ \dfrac{1}{2}x^{-1/2} + 1 = 1 + \dfrac{1}{\sqrt{x}}$ *	M1 B1 E1 [3]	Expanding bracket (correctly) $\dfrac{1}{2}x^{-1/2}$
(b) $2\log x + \log 3 = \log(3x^2)$ $\log(3x^2) = \log 75 \Rightarrow 3x^2 = 75$ $\Rightarrow x^2 = 25$ $\Rightarrow x = 5$	B1 B1 M1ft A1 cao [4]	x^2 ×3 (dep on 1st B1) anti-logging condone ±5
(c) $\displaystyle\int \frac{x}{(x^2-1)^3}dx$ let $u = x^2 - 1$, $du = 2x\,dx$ $= \displaystyle\int \frac{\frac{1}{2}}{u^3}du = \frac{1}{2}\int u^{-3}du$ $= \dfrac{1}{2}.\dfrac{u^{-2}}{(-2)} + c$ $= -\dfrac{1}{4(x^2-1)^2} + c$	M1 A1 A1ft A1 cao [4]	Substituting $u = x^2 - 1$ with an attempt to substitute for dx in terms of du. Correct integrand wrt u $-\dfrac{1}{4}u^{-2}$ By inspection: $\ldots(x^2-1)^{-2}$ M1 $k(x^2-1)^{-2}$ M1 ×-1/4 A2
(d) $\displaystyle\int_1^2 \frac{x^2+2}{x}dx = \int_1^2 (x + \frac{2}{x})dx$ $= \left[\dfrac{1}{2}x^2 + 2\ln x\right]_1^2$ $= 2 + 2\ln 2 - \dfrac{1}{2} = 2\ln 2 + \dfrac{3}{2} = 2.89$ or 2.9	M1 A1 M1 A1 cao [4] Total [15]	Splitting fraction correctly $\dfrac{1}{2}x^2 + 2\ln x$ Substituting limits consistent with their variable

2 (i) When $x = 0$, $y = -3$ so C is $(0, -3)$ When $y = 0$, $(x^2 - 3)e^{-x} = 0$ $\Rightarrow x^2 = 3$ $\Rightarrow x = \sqrt{3}$ or $-\sqrt{3}$ so A is $(-\sqrt{3}, 0)$ and D is $(\sqrt{3}, 0)$	B1 M1 B1, B1 [4]	 www
(ii) $y = (x^2 - 3)\,e^{-x}$ $\frac{dy}{dx} = (x^2 - 3)(-e^{-x}) + 2x.e^{-x}$ $= -e^{-x}(x^2 - 2x - 3)$ *	B1 B1 E1 [3]	$(x^2 - 3)(-e^{-x})+\ldots$ $\ldots + 2x.e^{-x}$ www
or $y = x^2 e^{-x} - 3\,e^{-x}$ $\Rightarrow \frac{dy}{dx} = 2x\,e^{-x} + x^2\,(-e^{-x}) - 3(-e^{-x})$ $= e^{-x}(2x - x^2 + 3)$ $= -\,e^{-x}(x^2 - 2x - 3)$	B1 B1 E1 [3]	$2x\,e^{-x} + \ldots$ $\ldots + x^2\,(-e^{-x})$ www
(iii) $\frac{dy}{dx} = 0$ when $x^2 - 2x - 3 = 0$ $\Rightarrow (x - 3)(x + 1) = 0$ $\Rightarrow x = 3, x = -1$* when $x = 3$, $y = 6e^{-3}$ or 0.3(0…) when $x = -1$, $y = -2e$ or $-5.4(4\ldots)$	M1 M1 E1 B1 B1 [5]	$x^2 - 2x - 3 = 0$ soi factorising or formula (correct) or M1 A1 A1 for verifying by substituting Mark final answer.
(iv) Both inflections correctly marked Inflections are when $\frac{d^2y}{dx^2} = 0$ $\Rightarrow x^2 - 4x - 1 = 0$ This has 2 (real) roots as discriminant $= 20 > 0$	B1 M1 E1 [3]	 Soi Or quadratic formula used correctly to demonstrate 2 distinct roots ($2 \pm \sqrt{5}$ or 4.24…, -0.24…)

3(a) (i) $t = 0 \Rightarrow h = 20 - 19 = 1$ metre	B1 [1]	
(ii) $h = 20 - 19 \times 0.9^2$ $= 4.61$ metres $10 = 20 - 19 \times 0.9^t$	 B1 M1	 4.6 or better
$\Rightarrow 0.9^t = \frac{20-10}{19} = \frac{10}{19}$ $\Rightarrow t \ln 0.9 = \ln(\frac{10}{19})$ $\Rightarrow t = \ln(\frac{10}{19}) / \ln 0.9 = 6.09$ years	A1 M1 A1	 taking logs or 6.1
or $10 = 19 \times 0.9^t$ $\ln 10 = \ln 19 + t \ln 0.9$ $t = (\ln 10 - \ln 19)/\ln 0.9$ $= 6.09$	 M1 A1 A1	
or $20 - 19 \times 0.9^6 = 9.9$ $20 - 19 \times 0.9^{6.1} = 10.01$ so $t = 6.1$	M1 A2 [5]	Trial and improvement with at least two trials shown. or better
(iii) $\ln y = \ln a + b \ln h$ $\ln a = -2 \Rightarrow a = e^{-2} = 0.135$ $b = \text{gradient} = \frac{3-(-2)}{2-0} = 2.5$ so $y = 0.135\ h^{2.5}$	M1 A1ft M1 B1 A1 ft [5]	$\ln a = -2$ $a = 0.135, 0.14$ b= gradient (soi) correct expression for gradient $b = 2.5 \pm 0.1$ *or* M1 A2 for correctly calculating b using 1 point *or* fitting with 2 points: M1: forming 2 equations M2: solving simultaneously A1, A1 a.r.t 0.14, 2.5 ± 0.1
(iv) $y = 100 \Rightarrow \ln y = 4.605$ $\Rightarrow \ln h = \frac{4.605+2}{2.5}$ $= 2.642$ $\Rightarrow h = 14.04$ $\Rightarrow t = \frac{\ln(\frac{20-14.04}{19})}{\ln 0.9}$ $= 11$ years	M1ft A1ft M1 A1ft [4]	$\ln 100 = -2 + 2.5 \ln h$ or $100 = 0.135\ h^{2.5}$ $\Rightarrow h^{2.5} = \frac{100}{0.135} = 740.74$ h consistent with their a and b Substituting their h in the $h - t$ equation and solving using logs (www) ft on values of $a = 0.14 \pm 0.005$ and $b = 2.5 \pm 0.1$ www

4(a) (i) $b + b + (b+2) + (b+2) + (b+4) + (b+4)$ $= 6b + 12 = 6(b + 2)$	E1 [1]	or sum formula with $n = 3$.
(ii) A.P. with $a = 2b$, $d = 4$ $\Rightarrow S_n = \frac{n}{2}(4b + [n-1]4)$ $= 2n(b + n - 1)$ *	M1 A1 E1 [3]	*or* $2 \times$ AP with $a = b$, $d = 2$ $\Rightarrow 2 \times \frac{n}{2}(2b + [n-1]2)$ www
(iii) $2n(5 + n - 1) = 570$ $\Rightarrow n^2 + 4n = 285$ $\Rightarrow n^2 + 4n - 285 = 0$ $\Rightarrow (n + 19)(n - 15) = 0$ $\Rightarrow n = 15$	M1 M1 A1 cao [3]	Equating to 570 Factorising, formula or trial and improvement
(b)(i) $u_1 = 3 \times 1.25^{-1} = 2.4$ $u_2 = 3 \times 1.25^{--2} = 1.92$ $u_3 = 3 \times 1.25^{--3} = 1.536$ $r = 1/1.25 = 0.8$	B1 B1 B1 B1 [4]	
(ii) $\sum_{k=1}^{20} u_k = \frac{2.4(1-0.8^{20})}{1-0.8}$ $= 11.86$	M1ft A1 cao [2]	Substituting their r into GP sum formula, ft their r and a 11.9 or better
(iii) $S_\infty = \frac{2.4}{1-0.8} = 12$	M1 A1 cao [2]	ft their r provided $\lvert r \rvert < 1$

Examiner’s Report

2602 Pure Mathematics 2

General Comments

The paper attracted the full range of marks, with many candidates scoring over 50 marks, and very few failing to achieve 10 marks. All questions proved to be accessible, and few candidates failed to attempt all four questions, with many candidates scoring well on the final part of question 4. As in previous years, although there were many excellent scripts, there was also a not insignificant minority from students who showed very little basic knowledge of the syllabus, and should not have been entered.

The main weaknesses were in calculus, especially the two integration questions in question 1. Poor algebra and notation were still in evidence. Some obviously good candidates lost marks for showing insufficient working when asked to show or prove results: we suggest that candidates should be advised to show too much working rather than too little when establishing results.

This was the second P2 paper in which question 1 was split into a number of independent parts. The intention of these questions is to provide relatively straightforward short questions on topics, and candidates in January scored well on this question. However, this situation was reversed on this paper, where question 1 was the least well done question on the paper!

Comments on Individual Questions

Q.1 (a) This question was well done in general, although weaker candidates made errors, either with the chain rule or differentiating $x^{1/2}$.

(b) This was one of the weaker parts. Log $3x$ was a very common error in the first part. Many candidates re-started when solving the equation – answers which were numerically correct (i.e. $x = 5$) from approximate working were condoned. The answer $x = -5$, although clearly wrong, was also condoned.

(c) Most candidates attempted the substitution, but quite a few failed to deal with the x in the integrand. They needed to attempt to substitute for dx to gain a method mark. Other substitutions are possible, albeit harder, and were of course accepted. Those candidates who got as far as $\int \frac{du}{2u^3}$ often made a slip in integrating from here. The lack of a constant of integration was condoned, but its routine absence was disappointing.

(d) This question was very poorly done. Only a minority of candidates split the fraction first, and even if this was done, many made errors in integrating from there. The majority of candidates tried an integration by substitution - let $x = u$ was not uncommon! Other candidates applied quotient rule (for differentiating), or integrating top and bottom of the fraction separately. Unsupported (or ill supported) correct answers gained zero.

(b) $\log(3x^2), x = 5$; (c) $-\frac{1}{4(x^2-1)^3} + c$; (d) $2 \ln 2 + 1.5 = 2.89$.

Q.2 The presence of e^{-x} in the function unsettled some candidates; but they were aided by the generous amount of help given in the question, and many candidates scored highly.

(i) Most candidates managed to find the coordinates of the points, but weaker scripts got the coordinates the wrong way round.

(ii) Generally, the product rule was well known and applied correctly. The most common error was omitting the negative sign in differentiating e^{-x}, followed by some fudging to arrive at the given result. We looked for consistency of working here – judicious use of brackets was needed for convincing solutions.

(iii) Some candidates introduced spurious solutions to $e^{-x} = 0$, but most solved the quadratic either by factorising or by formula, and evaluated the y–coordinates successfully. Wrong rounding was penalised here.

(iv) The idea of oblique inflections was not well known, and many good candidates lost a mark by failing to sketch these convincingly. Setting the second derivative to zero was generally good – most then solved the quadratic rather than use the discriminant to argue for two points of inflection. The most common misconception here was to apply the second derivative test to confirm the nature of the turning points, which was irrelevant.

(i) A is $(-\sqrt{3}, 0)$, C is $(0, -3)$, D is $(\sqrt{3}, 0)$; (iii) $(3, 6e^{-3})$, $(-1, -2e)$.

Q.3 Many candidates scored highly in this question, and even the weakest candidates picked up marks in the earlier parts.

(i) This was very well answered.

(ii) $h = 4.61$ was usually evaluated correctly, and most candidates gained an easy mark for equating h to 10. However, they then went off the rails by rearranging incorrectly or applying logarithms incorrectly. To gain full marks for trial and improvement methods, we wanted to see evidence of at least two trials. Whole number solutions such as 6 or 7 years were accepted.

(iii) Most candidates used the obvious method of calculating the gradient for b and equating the intercept to $\ln a$. Weaker students showed muddled thinking here, for example taking logs when calculating the gradient or failing to solve for a.

(iv) Variations in a and b give quite a wide range of solutions here. Answers to (iii) were followed through as far as calculating the value of h, but solutions had to be correct to achieve the final two marks.

(i) 1 metre, (ii) 4.61 metres, 6.09 years, (iii) $a = e^{-2} = 0.135$, $b = 2.5$, (iv) $h = 14.04$, $t = 11$ years.

Q.4 (a)(i) This was usually correctly answered.

(ii) Solutions based on wordy explanations of the given formula usually scored nothing. There is evidence of a lack of understanding of the meaning of 'show' from candidates who simply verified the formula for individual values of n. Some candidates fudged the multiplication by two implied by counting outward and return journeys.

(iii) This was often well done, with errors coming from simplifying the quadratic incorrectly or using the quadratic formula incorrectly. Trial and error methods were accepted.

(b)(i) This was a source of easy marks for all candidates. $r = 1.25$ was the commonest error.

(ii) There were many correct solutions. Some failed to get the formula right, or used the wrong value for a or r.

(iii) The formula was well known; no follow through was allowed for $r = 1.25$, for obvious reasons.

(a)(iii) $n = 15$; (b)(i) 2.4, 1.92, 1.536, $r = 0.8$, (ii) 11.86, (iii) 12.